Math 210 - Quiz 2 (Fall 2010)

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1. Consider the following:

i- (15 points) Suppose $f(x)$ is a continous function on $[0,1]$, such that $\sup_{[0,1]}(f) = M$. Assume $\int_0^1 f(x)dx = M$. Prove that $f(x) = M \, , \forall x \in [0,1]$.

ii- (15 points) Suppose $f(x)$ is a continous function on $[0,2]$, such that $\sup_{[0,2]}(f) = M$. Assume $\int_0^2 f(x)dx = 2M$. Prove that $f(x) = M \, , \forall x \in [0,2]$.

2. Consider the following:

i- (15 points) Prove that a continuous function on a compact set is uniformly continuous.

ii- (15 points) Assume $f(x)$ is a function differentiable everywhere on $\mathbb{R}$, and that $|f'(x)| \leq A \, , \forall x$. Prove that f is uniformly continuous.

3. (20 points) Assume $f(x)$ is a function which is three times differentiable (i.e. $f'''(x)$ exists at every x). Moreover, you are given that $f(0) = f'(0) = f''(0) = 1$ and that $|f'''(x)| \leq 1 \, , \forall x$. Find (with careful justification) the following limit

$$\lim_{x \to 0} \frac{f(x) - 1 - x}{x^2}$$

4. (20 points) Suppose f and g are differentiable on $[a,b]$ and that f' and g' are Riemann integrable on $[a,b]$. Prove the following formula:

$$\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx$$

The fact that the above integrals exist should be proven as well.